Nonreciprocal Dispersion Characteristics of a Planar Helix on Magnetized Ferrite Slabs

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***Abstract*—A structure comprising a pair unidirectionally conducting (UC) screens on a pair of parallel magnetized ferrite slabs is considered. The screens are conducting in different directions, thus constituting a planar helix. The nonreciprocal dispersion characteristics of the structure are studied. The slow-wave properties of the structure coupled with the fact that the guided waves on it are, in general, elliptically polarized results in relatively large differential phase shifts per unit length. Possible use of the structure as a phase shifter in planar configuration is indicated.**

I. Introduction

A SLOW-WAVE structure in planar geometry, consisting of a pair of parallel unidirectionally conducting (UC) screens conducting in different directions and separated by some distance in air, was studied by Arora [1], [2]. The structure was generalized in [3] to the case where the media between the two screens and in the outer regions were different dielectrics, and possible application to nonreciprocal ferrite devices was suggested. This suggestion is pursued here somewhat further. A study of the field configuration and polarization properties of the structure described in [3] is undertaken. Since it is indicated that the field configuration is suitable for nonreciprocal interaction, the dispersion characteristics are obtained for the case when a pair of ferrite slabs is inserted between the two UC screens. The medium between the slabs and the outer medium are assumed to be dielectrics. The two ferrite slabs are magnetized to saturation in opposite directions, and the change in dispersion consequent upon a reversal of the direction of magnetization is studied.

II. Field Configuration in a Planar Helix

Before considering the case of a ferrite-filled planar helix, it is helpful to visualize the nature of the field in a planar helix on a dielectric, such as the one reported in [3]. As shown in Fig. 1, the top and the bottom screens conduct in directions y' and y'' which make angles α and $-\alpha$, respectively, with the y axis. As pointed out in [3], for all the modes $E_{y'}$ vanishes in the entire region $x \geqslant a$ and $E_{y''}$ vanishes in the entire region $x \leqslant -a$. In the sandwiched region, the electric field vanishes in directions

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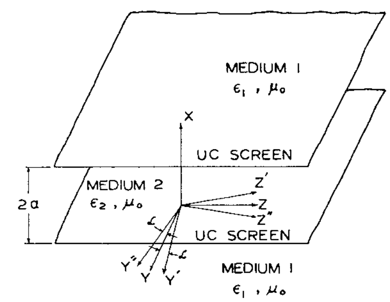

Fig. 1. The planar helix, z is direction of propagation; y',y'' are directions of conduction of the top and bottom UC screens, respectively; α is helix angle.

making angles θ_s and θ_a with the y axis for symmetric and antisymmetric modes, respectively, where these angles are given by

$$\begin{matrix}\theta_s \\ \theta_a\end{matrix} = \tan^{-1}\left(\tan\alpha \begin{matrix}\tanh \\ \coth\end{matrix} u_2 a \begin{matrix}\coth \\ \tanh\end{matrix} u_2 x\right). \tag{1}$$

The direction of vanishing magnetic-field component also rotates in a similar fashion, but, in general, this direction is different from the direction of vanishing electric-field component. At any fixed point, the electric- and magnetic-field components in the planes, which are perpendicular to the directions in which the respective components vanish, are out of phase with respect to each other by a quarter-cycle, which means that both electric and magnetic fields are elliptically polarized. Fig. 2(a) and (b) depict the instantaneous field for modes 1 and 2 of the "normal helix" for the particular case $\epsilon_{r1}=1$, $\epsilon_{r2}=2.56$, $\alpha=30$ degrees, and $k_0a=0.6$. (The notations used here are as in [3].) The sense of rotation of the field vectors in different regions of the structure is indicated in the figures.

III. Planar Helix on Ferrite Slabs

The elliptical polarization of the fields readily suggests that the above structure may be modified to yield nonreciprocal characteristics if the dielectric between the screens is replaced by a magnetized ferrite. To explore this possibility in some detail, the structure indicated in Fig. 3 is considered. A pair of ferrite slabs is sandwiched by a pair of UC screens. The structure is assumed to be of infinite width in the y direction with the regions between the ferrite slabs and outside the UC screens filled with

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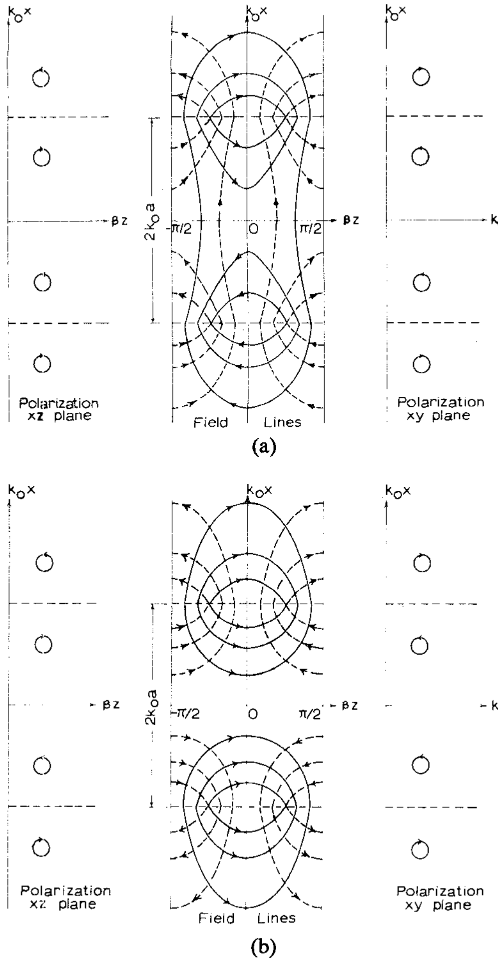

Fig. 2. Field configuration and polarization of two of the modes of a planar normal helix. Electric-field lines are shown continuous and magnetic-field lines are shown dashed. The curves are plotted for $\alpha=30$ degrees and $k_0a=0.6$. Fig. 2(a) and (b) relate, respectively, to modes indicated by numbers 1 and 2 in fig. 2 of [3].

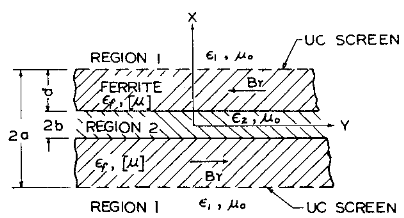

Fig. 3. Configuration considered for theoretical analysis. The structure is of infinite extent in y and z directions. The top and bottom UC screens conduct in directions making angles α and $-\alpha$, respectively, with the y axis.

different dielectrics, in general. It is also assumed that the ferrite slabs are in the state of remanent magnetization, in opposite directions, as indicated in Fig. 3. In practice, such magnetization may be achieved using a ferrite toroid with a magnetizing wire running longitudinally. The geometry of the structure is taken to be symmetrical about the $x=0$ plane.

IV. Characteristic Equation

As in [3], the analysis is carried out in the sheath-helix approximation. The top and bottom screens are assumed to conduct in the y' and y'' directions which make angles α and $-\alpha$, respectively, with the y axis. Also, using the symmetry of the structure, the general solution is separated into two types of modes: 1) transverse-symmetric (even), and 2) transverse-antisymmetric (odd). In the former type of modes, field components E_x and E_y have a maximum at $x=0$, while in the latter type of modes, these components pass through a zero at $x=0$. Symmetry also ensures that only one-half of the structure need be considered.

The analysis is carried out by the impedance-transformation method. Viewing the structure as a transmission line along the x direction, the following relations hold for the transverse-symmetric case:

$$H_z(0)/E_y(0)=0; \qquad H_y(0)/E_z(0)=\pm\infty \tag{2}$$

$$H_z(b+0)/E_y(b+0)=H_z(b-0)/E_y(b-0)$$
$$H_y(b+0)/E_z(b+0)=H_y(b-0)/E_z(b-0) \tag{3}$$

$$H_y(a+0)/E_z(a+0)=-j\omega\epsilon_1/u_1$$
$$H_z(a+0)/E_y(a+0)=u_1/j\omega\mu_0 \tag{4}$$

where $u_1^2=\beta^2-\omega^2\mu_0\epsilon_1$, β being the phase constant in the z direction, and ϵ_1 being the permittivity of region 1. Finally, across $x=a$

$$\left[\frac{H_z(a+0)}{E_y(a+0)}-\frac{H_z(a-0)}{E_y(a-0)}\right]\tan^2\alpha = \left[\frac{H_y(a+0)}{E_z(a+0)}-\frac{H_y(a-0)}{E_z(a-0)}\right] \tag{5}$$

which equation incorporates the boundary conditions at the UC screen.

The above equations result in the following characteristic equation for transverse-symmetric modes:

$$\tan^2\alpha\left[1+\frac{u_2}{u_1}\,\frac{\tanh u_2b\left(1+\dfrac{\beta}{v_1}\dfrac{K}{\mu}\tanh v_1d\right)+\dfrac{\mu_0}{\mu_{\text{eff}}}\dfrac{v_1}{u_2}\left(1-\dfrac{K^2}{\mu^2}\dfrac{\beta^2}{v_1^2}\right)\tanh v_1d}{1+\dfrac{u_2}{v_1}\tanh v_1d\left(\dfrac{\mu_{\text{eff}}}{\mu_0}\tanh u_2b-\dfrac{K}{\mu}\dfrac{\beta}{u_2}\right)}\right] = \frac{k_0^2}{u_1^2}\left[\epsilon_{r1}+\frac{\epsilon_{rf}u_1}{v_2}\,\frac{1+\dfrac{\epsilon_{rf}u_2}{\epsilon_{r2}v_2}\tanh u_2b\tanh v_2d}{\tanh v_2d+\dfrac{u_2\epsilon_{rf}}{\epsilon_{r2}v_2}\tanh u_2b}\right]. \tag{6}$$

For transverse-antisymmetric modes, (2) changes to

$$H_z(0)/E_y(0) = \pm\infty \qquad H_y(0)/E_z(0) = 0 \tag{7}$$

while the relations (3)–(5) remain unchanged. It turns out that the characteristic equation in this case can be written simply by changing $\tanh u_2 b$ to $\coth u_2 b$ in (6). The symbols introduced in (6) have the following meanings:

$$k_0^2 = \omega^2 \mu_0 \epsilon_0$$
$$v_1^2 = \beta^2 - \omega^2 \mu_{\text{eff}} \epsilon_f$$
$$v_2^2 = \beta^2 - \omega^2 \mu_0 \epsilon_f$$
$$u_2^2 = \beta^2 - \omega^2 \mu_0 \epsilon_2 \tag{8}$$

where ϵ_2 and ϵ_f are the permittivities of region 2 and ferrite, respectively, and ϵ_r's denote the relative permittivities of the respective regions. The quantities μ, K, and μ_{eff} relate to ferrite and carry the usual meaning (see, for example, [4]):

$$\mu = \mu_0 \left[1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \right]$$
$$K = \mu_0 \frac{\omega \omega_m}{\omega_0^2 - \omega^2}$$

and

$$\mu_{\text{eff}} = (\mu^2 - K^2)/\mu. \tag{9}$$

V. Dispersion Characteristics

For numerical solution of the characteristic equations, the following values are assumed for different parameters: $4\pi M_r = 1300$ G, $\epsilon_{rf} = 13.0$, $b/d = 0.6$, $\alpha = 30$ degrees, and $\epsilon_{r1} = \epsilon_{r2} = 1$. Fig. 4(a) gives the dispersion curves for transverse-symmetric modes, while similar curves for transverse-antisymmetric modes are given in Fig. 4(b). The quantities β^+ and β^- denote the phase constants for opposite directions of magnetization. As in [3], it is observed that there are two modes of the symmetric type (modes 1 and 3) and one mode of the antisymmetric type (mode 2) which propagate from zero frequency onwards. Modes 1 and 2 can be identified as the "screen modes", inasmuch as these modes are retained even in the absence of the ferrite slabs. It is worth noting that these modes are slower than the others. Therefore, as may be expected, $|\beta^+ d - \beta^- d|$ is also the highest for these modes. In addition to the above modes, higher order modes of both symmetric and antisymmetric types exist. For the present case, when no dielectric is considered, the cutoff points for higher order symmetric modes are given by

$$k_0 d = (n\pi/2)\left[(\mu_{\text{eff}}/\mu_0)\epsilon_{rf} - 1 \right]^{-1/2}, \qquad n = 1, 2, \cdots. \tag{10}$$

For higher order antisymmetric modes, there occur two

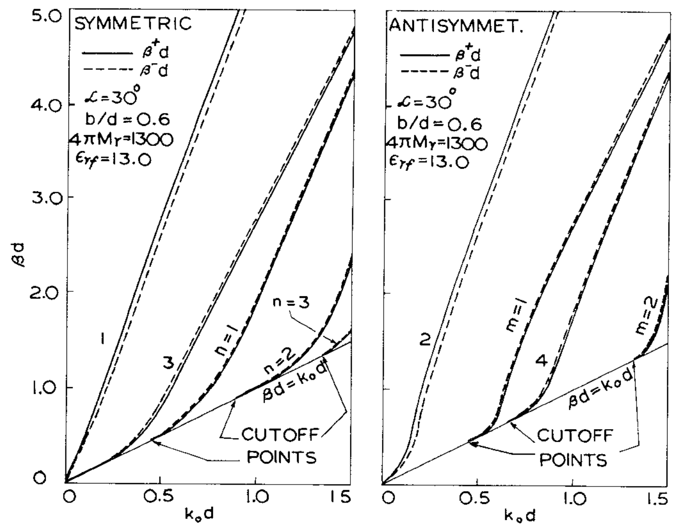

Fig. 4. Dispersion characteristics for the configuration of Fig. 3. $\epsilon_{r1} = \epsilon_{r2} = 1.0$, $\alpha = 30$ degrees, $\epsilon_{rf} = 13.0$, $4\pi M_r = 1300$ G, and $b/d = 0.6$. Continuous and dashed curves represent the characteristics for two opposite senses of magnetization of ferrite slabs. (a) Transverse-symmetric modes. (b) Transverse-antisymmetric modes.

sets of cutoff points. One of these sets is given by

$$k_0 d = \left[(2m-1)\pi/2 \right]\left[(\mu_{\text{eff}}/\mu_0)\epsilon_{rf} - 1 \right]^{-1/2}, \qquad m = 1, 2, \cdots \tag{11}$$

while for the other one

$$k_0 d = w_1 d\left[(\mu_{\text{eff}}/\mu_0)\epsilon_{rf} - 1 \right]^{-1/2} \tag{12}$$

where $w_1 d = j v_1 d$ represents the solutions of

$$\tan w_1 d = -w_1 b / \left[(\mu_{\text{eff}}/\mu_0) - (K/\mu) k_0 b \right]. \tag{13}$$

A few of the lowest order solutions of (13) are $w_1 d = 2.215$, 5.032, 8.05. Mode 4 in Fig. 4(b) corresponds to the latter set of cutoff points.

Unimodal propagation can be achieved by suitable excitation, since there exists only one antisymmetric mode which propagates down to zero frequency.

Dispersion curves similar to those in Fig. 4 can be used to compute the maximum differential phase shift as a function of frequency for a structure of given dimensions. Results so obtained are given in Fig. 5 for $d = 1$ mm, and $b = 0.6$ mm and 1.5 mm. Modes 1 and 2 only are considered.

The antisymmetric mode is seen to result in a higher phase shift. Phase shift in excess of 250 degrees per inch is indicated for the entire X band. In general, the phase shift is higher for a smaller gap between the ferrite slabs. This is understandable, since the screens interact more strongly as the separation between them decreases; this also leads to a higher degree of slowing down. It is of interest from a practical point of view to note that the differential phase shift may be rendered insensitive to frequency over a wide band by a suitable design, as is indicated by the curve for the antisymmetric mode when $b = 1.5$ mm.

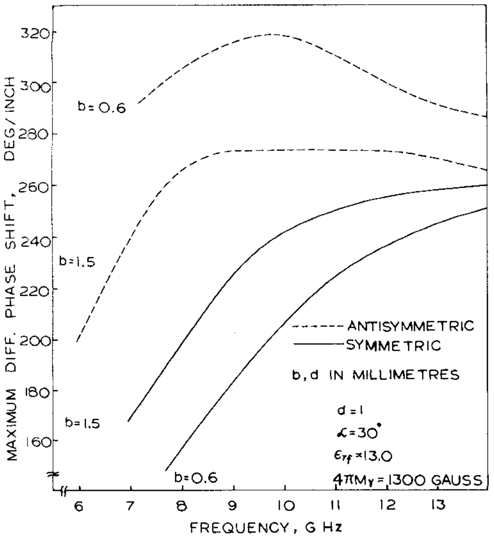

Fig. 5. Phase shift versus frequency for $d=1$ mm for two different separations of ferrite slabs, namely, $b=0.6$ mm and 1.5 mm. Other parameters have values identical to those for Fig. 4.

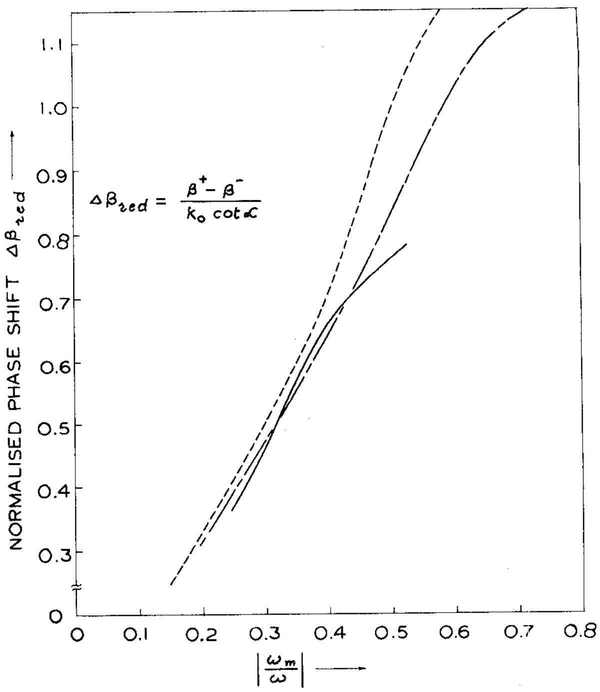

Fig. 6. Normalized differential phase shift coefficient versus remanent magnetization for the planar helix and circular helix. Results for the circular helix are taken from [6] and [8]; —— normal planar helix, antisymmetric mode, $b=0.6$ mm, $d=1.0$ mm, $\epsilon_{rf}=13.0$; ----- normal circular helix, $r_{\text{red}}=r_0k_0 \cot\alpha=1$ (r_0=helix radius), $\epsilon_{rf}=12.0$ ([6], fig. 3); —·— normal circular helix, $r_{\text{red}}=1$, $\epsilon_{rf}=10.0$ ([8], fig. 3).

VI. Discussion

It has been pointed out elsewhere [3], [5] that the slow-wave structure used here resembles a circular helix in its electromagnetic guiding properties. Nonreciprocal propagation characteristics have been obtained in the cases of the circular helix [6]–[9] and other periodic slow-wave structures [10]–[12] loaded with ferrite. It should be noted here that planar slow-wave structures of the meander-line type suffer from an inherent limitation on their bandwidth because of the presence of periodic right-angle bends [13]; typical bandwidths are around 10 percent [14]. The present structure, on the other hand, is essentially a wide-band structure.

The case considered here, in which the helix is outside the ferrite, has been referred to as the normal helix [3]. It is reasonable to expect some correspondence with the normal circular helix. To facilitate this comparison, phase shifts for the planar and the circular helices are plotted together in Fig. 6. Results for the circular helix are taken from [6] and [8]. In the region $|\omega_m/\omega|<0.5$, the phase shifts are seen to be comparable.

The results presented here are based on the sheath-helix approximation. A more rigorous analysis, on the lines of the tape-helix model introduced by Sensiper [15], would be complicated because of the presence of magnetized ferrite. In any case, the sheath-helix approximation has been in use even for the circular helix and is known to yield realistic estimates of phase shift.

In a preliminary experiment, closely spaced narrow conducting lines printed on alumina substrates (50.8 mm $\times$25.4 mm$\times$0.64 mm) were stuck with the screens on the outer sides to the top and bottom of a 50.8-mm long and 25.4-mm wide four-piece ferrite toroid with $2b=d=1.6$ mm. Material parameters of ferrite were $4\pi M_s=1700$ gauss and $\epsilon_{rf}=13.0$. A number of turns of a latching wire were threaded through the ferrite toroid. At 10 GHz, this arrangement resulted in a maximum differential phase shift of 300 degrees for $\alpha=30$ degrees. The phase shift can be increased by printing the UC screens directly on the ferrite. A comparison with theory may not be very meaningful, since in the analysis the structure width is assumed to be infinite. Attempts are under way to carry out an analysis taking into account a finite width of the planar helix.

From a practical point of view, an aspect of vital importance is that of excitation of the planar helix. A study of the excitation problem for a planar helix using air as dielectric revealed that either the symmetric or antisymmetric mode can be excited using line sources in appropriate configurations [2]. One possible experimental arrangement for exciting the antisymmetric mode was suggested by Ash [16]. A symmetric mode, on the other hand, can be excited using a microstrip. As the helix angle α approaches $\pi/2$, the planar helix tends to support a field configuration approaching TM, the propagating wave comprising E_x, H_y, and E_z [17]. By gradually flaring out the top conductor, a microstrip may be modified to obtain a parallel-plate waveguide. Such a waveguide may then be interfaced with a planar helix of $\alpha=\pi/2$. Finally, α may be reduced gradually to the required value.

VII. Conclusion

It is shown here that a pair of parallel unidirectionally conducting screens with ferrite can be made use of in obtaining nonreciprocal propagation characteristics. Thus, a new configuration is indicated for possible use in a ferrite phase shifter. Dispersion characteristics of this configuration indicate a maximum differential phase shift greater than 250 degrees per inch at X band. Of course, much computational as well as experimental work remains to be done before a working phase shifter can be constructed on this principle.

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